ORIGINAL RESEARCH

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Examination of "The Tide," CRI, and Physiologic Respiration: A Mathematical Analysis

Abstract

This study seeks to examine the inter-relationship of what William Garner Sutherland, DO called "The Tide," the Cranial Rhythmic Impulse (CRI) palpated by practitioners of Osteopathy in the Cranial Field (OCF), and physiologic or pulmonary respiration. This study is different in that it proposes a mathematical analysis of these several phenomena to reconcile whether these are part of the same overarching natural wave or they are distinct. The pertinent medical literature is reviewed. The mathematics of Fourier analysis is briefly described. This study performed "reverse" Fourier analysis, that is, the combination of the various periodic signals into one periodic signal, across the range of normal physiological data values. Stochastic techniques were then applied to resolve these probabilistic data into 1 overarching periodic signal. From the mathematical analysis, it is not mathematically possible for "The Tide" to be in the range of the fundamental frequencies of the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations that derive from it and are its harmonics. This mathematical analysis cannot be construed as negating the existence of "The Tide," rather merely eliminates these fundamental frequencies as being "The Tide." "The Tide" must be something else. More research is indicated.

Purpose

This study seeks to examine the inter-relationship of what William Garner Sutherland, DO called "The Tide," the Cranial Rhythmic Impulse (CRI) palpated by practitioners of Osteopathy in the Cranial Field (OCF), and physiologic or pulmonary respiration.[1,2] This study is different in that it proposes a mathematical analysis of these several phenomena to reconcile whether these are part of the same overarching natural wave or they are distinct.

Survey of the Literature

The living human body exhibits several rhythmic, cyclic phenomena. Examples including the heartbeat, pulses, and pulmonary respiration are well-known to not only health professionals, scientists, and students of the life sciences, but also to the general public. In his book, *Osteopathy in the Cranial Field*, Harold Ives Magoun, DO, FAAO wrote:

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Editor's Note

This article was completed as part of the requirement toward earning the designation of Fellow in the American Academy of Osteopathy. Opinions expressed in this article are those of the author and do not necessarily reflect the viewpoint or official policy of the American Academy of Osteopathy, and it was edited to conform to *AAOJ* style guidelines.

Keywords

Cranial Rhythmic Impulse (CRI), Osteopathy in the Cranial Field (OCF), Osteopathic Cranial Manipulative Medicine (OCMM), Physiologic Respiration, The Tide, Mathematical Analysis, Traube-Hering-Mayer oscillations

Four definite motions have been observed at operation:

1. A pulsation which is synchronous with cardiac contractions.
2. A pulsation which coincides with respiratory pressure changes associated with inhalation and exhalation.
3. A wave not related to either heart rate or respiration but one which constantly maintains its own cycle.
4. An undulating pulsation which has not been identified.[3]

Nelson and associates presented an explanation of the differences between palpated and instrumentally recorded rates for the CRI.[4,5] Moskalenko and associates recently proposed a liquorodynamic model of the Primary Respiratory Mechanism (PRM) that "does not contradict previously published papers, and, more importantly, adheres to the laws of biomechanics."[6]

Masiello wrote a treatise of "The Tide" from a philosophical perspective.[7] His paper placed the writings of Still, Sutherland, and Wales into philosophical context based on the writings of several 20th-century philosophers. This helped put the earlier writings of both Still[8,9,10] and Sutherland[1] into greater perspective and brought about increased understanding; however, it is not from the greater philosophical concept of "The Tide" that this study attempted to consider it. Rather, it is from the biomechanical and physiological perspective that "The Tide" is considered. Citing Becker, Nelson reported a "fast tide" and a "slow tide" as components of the CRI. Nelson further stated that the "fast tide" component of the CRI was in the range of 8-12 cycles per minute (cpm), while the "slow tide" component was approximately 0.6 cpm.[11] These values are consistent with the range of Ferguson (0.6-15 cpm)[12] below.

There has been argument about whether the terms "Cranial Rhythmic Impulse" and "Primary Respiratory Mechanism" are synonymous. In his paper, "Neurobiological aspects of the cranial rhythmic impulse," Schleip states they are the same.[13] Meanwhile, Pribadi discussed palpating the CRI throughout the body,[14,15] while Lee reinforces the specificity of the terminology stating,

> the term, "CRI" should be restricted to the situation where one is measuring the rate or amplitude of the primary respiratory mechanism in the head for research purposes as John and Rachel Woods used it when they coined the term doing their research…the applicable term is primary respiratory mechanism or primary respiration. Other terms such as cranial rhythm are less desirable because they are less generalizable. Primary respiration and primary respiratory mechanism are fully generalizable to the whole body. These terms describe the physiology that leads to metabolism in the whole organism.

For the purposes of this study, the CRI is considered in the sense of Lee and what is palpated in the head, thus obviating the argument. In his paper, "The Inherent Rhythmic Motion of Cranial Bones," Hollis King, DO, PhD, FAAO, discussed the various experimental research by both the United States and Russian (then Soviet) space agencies which demonstrated the movement of the cranial bones.[17]

The rate of the CRI varies. Magoun stated the CRI to have a rate of 10-14 cycles per minute.[3(pp25,40)] Ferguson stated the CRI rate as being 6-15 cycles per minute.[12] Ferguson also reported slower rates. In particular, he stated, "Norton et al. reported a rate of 3.7 cpm average… and Becker reported a 'deeper rhythm' of 0.6 cpm."[16(p75)] Ferguson went on to discuss the "rhythmic physical motion of the tissues of the cranium in the range of 0.6 to 15 cpm, which are the limits of the reported range of CRI palpated."[16(p75)] Nelson initially stated, "The CRI has a traditionally agreed upon rate of 10 to 14 cycles per minute." He then goes on to state, "rates have been reported for the CRI of between 3 and 14 cycles/min."[11] Nelson also reported experimental palpation rate of 4.54 cpm.[4] Nelson finally reported, "A new normative range for the CRI of 2-7 cpm, as palpated by experienced examiners, has been identified."[18]

Together, the Traube-Hering-Mayer oscillations have been known since the latter half of the 19th century. Nelson and associates demonstrated some simultaneity between the CRI and Traube-Hering-Mayer oscillations by comparing laser-Doppler flowmetry and palpation. Their work compares the CRI with flows and oscillations found within the vasculature. They stopped short of claiming the CRI and the Traube-Hering-Mayer oscillations to be one and the same. Rather, they stated, "The [Traube-Hering-Mayer] oscillations may well represent one aspect of the complex arena of Sutherland's discovery. It may explain the rate and rhythm of the CRI and offer insight into the physiologic mechanism of the PRM."[19] Of note, McFarland and Mein discussed the CRI being the palpable result the integration of oscillations (that is, "entrainment") and is therefore a "harmonic frequency that incorporates the rhythms of multiple biological oscillators" (for example, pulmonary respiration heart rate, etc.).[20]

Moskalenko and Kravchenko discussed frequency fluctuations as being associated with the PRM. In particular, they presented a discussion of spectral analysis which they related to "slow fluctuations" of the CRI. They then compared variations in the PRM with changes in cardiovascular and respiratory systems that they associated with the performance of OCF.[21] Later, Moskalenko and associates subsumed this and other frequency analysis studies[22,23] into a "liquorodynamic model" of the PRM based upon frequency analysis – that is, Fourier analysis.[6]

Normal respiratory rate varies; the medical literature is often lacking. "Standard" rate is 20 breaths per minute.[24] According to Ganong's Review of Medical Physiology (24th ed.), the respiratory rate for normal adults varies from 12-20 breaths per minute.[25] Cretikos and associates stated that as recently as 2007 respiratory rate was often overlooked when taking vital signs and that adult respiratory rates of greater than 20 breaths per minute were associated with poor health outcomes.[26] Yuan and associates published a clinical review showing that breathing rates of 18-20 breaths per minute in adults were strongly associated (OR = 3.93 to 5.56) with cardiopulmonary arrest within 72 hours and/or death within 30 days.[27] While recent medical literature about normal adult respiratory rates is lacking, Fleming and associates published a systematic review of observational studies regarding pediatric respiratory and heart rates. Of note, they cited the following sources of adolescent (ages 13-18) respiratory rates: the UK Advanced Support Life Group (15-20 breaths per minute), American Heart association Pediatric Advanced Life Support Provider Manual (12-16 breaths per minute), and the European Resuscitation Council (12-20 breaths per minute).[28] These data are consistent with Ganong above; accordingly, for this study, normal adult respiratory rate will be 12-20 breaths per minute.

Review of Mathematical Techniques

In mathematics, it is well known that all waveforms (or signals) displaying some apparent periodicity or cyclic characteristics are the combination of sinusoidal equations called periodic functions. Furthermore, each periodic function can be broken down into its simpler, constituent sinusoidal component functions by application of the mathematical technique known as Fourier analysis. The result of this decomposition is known as the Fourier series. Integral multiples of the fundamental frequencies of any signal are known as "harmonics."

Clinicians are familiar with a variety of waveforms (signals) found in living beings. The pulse and respiration are things in the everyday experience of physicians and other health care providers. The inhalation-exhalation "cycle" of respiration is a simple example of a periodic waveform (as visualized on a display monitor). The various waveforms observed in an electrocardiogram (ECG) are examples of more complex periodic waveforms. To help understand the analysis of the several waveforms (signals) that are found in living beings, it is appropriate to first explain the basic principles of the underlying mathematics.

In secondary school mathematics, we learned the definitions of common "functions". For example,

(1) $y = f(x) = mx + b$

is the "standard" form of a straight line.

(2) $y = f(x) = ax2 + bx + c$

is the standard for of a parabolic curve. To put these into perspective, one needs to recall the set of x and y axes (horizontal and vertical, respectively) that form the basis for the graph of these functions. Equation (1) above is interpreted to mean:

The dependent variable, "y," is a function of the independent variable, "x," which is defined as a straight line with "slope" of "m" (the change of y divided by the change of x) and which crosses the vertical axis at value "b."

Similarly, equation (2) is interpreted to mean:

The dependent variable, "y," is a function of the independent variable, "x," which is defined as a curve formed symmetrically around the vertical axis and which crosses the vertical axis at value "c."

There are more complex, generalized forms of many functions.

A function is "periodic" if its waveform (signal) repeats with the same period, such that

(3) $y = f(t) = f(t + P)$

Which means that the dependent variable, "y," is a function of the independent variable, "t" (typically, "time"), that repeats exactly every period, "P." In high school trigonometry, we learned the simplest periodic waveform, that of the sinusoidal function given by

(4) $y = f(t) = \sin(t)$

The general form of a trigonometric function is the "trigonometric series," given by the equation:

$$y = f(t) = A0 + \sum_{n=1}^{\infty} [An \cos(nt) + Bn \sin(nt)] \quad (5)$$

In 1822, Jean-Baptiste Joseph Fourier published The Analytical Theory of Heat, which contained his work in the area of trigonometric series wherein he developed and demonstrated the concept that all periodic waveforms are made up of, and can be broken down into, simpler sinusoidal waveforms. This is given by the general form:

$$y = f(t) = sN(t) = A0/2 + \sum_{n=1}^{N} An \sin[(2\pi nt) + \phi] \quad (6)$$

Equation (6) above is known as the Fourier series in his honor. Fourier series forms the foundations for Fourier analysis, which is the study of the manner in which general functions can be represented by sets of trigonometric functions. The process by which these sets of trigonometric functions are broken down into the simpler sinusoidal waveforms is the "Fourier transform."

Early work by mathematician Frank Irwin of the University of California (Berkeley) demonstrated convergence of the series derived from "harmonics," that is the simpler sinusoidal waveforms. Later work by Lo demonstrated the ability for an "inverse operator" to be constructed (that is, "reverse" Fourier analysis), which allows for the simpler sinusoidal waveforms to reconstitute the more complex waveform. For those with a significant mathematical or engineering background, Boashash provides a comprehensive discussion and review of the above equations.[32,33]

Since periodic signals observed in living beings have a "range" within which the data values are considered normal, the actual value that may be expected is probabilistic. The application of deterministic mathematical models does not apply in these instances. Rather, the application of more complex stochastic techniques will be required. Fortunately, the laborious computational process of this analysis is obviated by computer tools which are freely available online.

Method

This study performed "reverse" Fourier analysis, that is, the combination of the various periodic signals into one periodic signal, across the range of normal physiological data values. Stochastic techniques were then applied to resolve these probabilistic data into one overarching periodic signal.

In order to accomplish this, a set of sinusoidal functions were first determined to model the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations in the very-low-frequency, low-frequency, and high-frequency ranges, respectively, as the terms in the Fourier series. The sinusoidal functions took the general form:

(7) $y = f(t) = \sin(\omega 1t) + \sin(\omega 2t) + \sin(\omega 3t)$

where $\omega 1$ = the CRI, $\omega 2$ = pulmonary respiration rate, and $\omega 3$ = the Traube-Hering-Mayer oscillations. The fundamental frequency, $\omega 0$, was computed for each function. These fundamental frequencies were then compared with data reported by Magoun and Nelson.

Analysis of the Data

The first function which serves to model the lowest CRI, pulmonary respiration rate, and lowest Traube-Hering-Mayer oscillations in the low-frequency range is given by equation (8) below:

(8) $y1 = f1(t) = \sin(\omega 11t) + \sin(\omega 21t) + \sin(\omega 31t)$

$y1 = f1(t) = \sin(0.1t) + \sin(0.2t) + \sin(0.1t)$

solving for the first fundamental frequency, $\omega 01$ = 1Hz

where Hz = Hertz = cycles per second

The second function which serves to model the highest CRI, pulmonary respiration rate, and highest Traube-Hering-Mayer oscillations in the low-frequency range is given by equation (9) below:

(9) $y2 = f2(t) = \sin(\omega 12t) + \sin(\omega 22t) + \sin(\omega 32t)$

$y2 = f2(t) = \sin(0.1t) + \sin(0.2t) + \sin(0.1t)$

solving for the second fundamental frequency, $\omega 02$ = 0.2Hz

The third and fourth functions serve to model the lowest and highest CRI, pulmonary respiration rate, and Traube-Hering-Mayer oscillations, respectively, in the very-low-frequency range and are given by equations (10) and (11) below:

Table 1. *Converting Hz to cycles per minute, summary.*

	VLF range	LF range	HF range
Lowest CRI, pulmonary respiratory rate, and Hering-Traube-Mayer oscillations	0.2Hz 12cpm	1Hz 60cpm	0.1Hz 6cpm
Highest CRI, pulmonary respiratory rate, and Hering-Traube-Mayer oscillations	0.2Hz 12cpm	0.2Hz 12cpm	0.5Hz 30cpm

(10) $y3 = f3(t) = \sin(\omega 13t) + \sin(\omega 23t) + \sin(\omega 33t)$

$y3 = f3(t) = \sin(0.003t) + \sin(0.2t) + \sin(0.1t)$

solving for the third fundamental frequency, $\omega 03$ = 0.2Hz, and

(11) $y4 = f4(t) = \sin(\omega 14t) + \sin(\omega 24t) + \sin(\omega 34t)$

$y4 = f4(t) = \sin(0.003t) + \sin(0.2t) + \sin(0.2t)$

solving for the fourth fundamental frequency, $\omega 04$ = 0.2Hz

Finally, the fifth and sixth functions serve to model the lowest and highest CRI, pulmonary respiration rate, and Traube-Hering-Mayer oscillations, respectively, in the high-frequency range and are given by equations (12) and (13) below:

(12) $y5 = f5(t) = \sin(\omega 15t) + \sin(\omega 25t) + \sin(\omega 35t)$

$y5 = f5(t) = \sin(0.25t) + \sin(0.2t) + \sin(0.1t)$

solving for the fifth fundamental frequency, $\omega 05$ = 0.1Hz, and

(13) $y6 = f6(t) = \sin(\omega 16t) + \sin(\omega 26t) + \sin(\omega 36t)$

$y6 = f6(t) = \sin(0.5t) + \sin(0.2t) + \sin(0.2t)$

solving for the sixth fundamental frequency, $\omega 06$ = 0.5Hz

Converting Hz to cycles per minute (cpm) and summarizing, we find the results in Table 1.

Discussion

The cranial concept within the history of osteopathy and osteopathic medicine has not been universally accepted. The presence of “The Tide” has not been universally palpated by those who apply the cranial concept to the treatment of their patients. This study does not have as its purpose to either prove or disprove, nor add to the data or arguments regarding these. Rather, these have been eloquently argued elsewhere in the literature and are to be taken as postulated or axiomatic. Central to this study is consideration of whether “The Tide” subsumes the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations – that is, that the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations are harmonic sinusoidal functions (in the mathematical sense) that derive from “The Tide.”

The application of mathematical analyses, in particular Fourier analysis, to biological and medical investigation has long been established. D’Avenio and associates demonstrated the application of Fourier transforms in identifying proteins when provided with unknown genome sequences of microorganisms.[34] Rohde and associates successfully applied Fourier analysis to distinguish differences in shape of unruptured vs. ruptured intracranial aneurysms.[35] Takalo and associates applied Fourier analysis to the study of circadian rhythms of the low-frequency oscillations and demonstrated the association of frequency shift of Mayer waves to lower frequencies with increased risk of developing hypertension.[36] Subsequent research by Nelson and associates applied Fourier analysis to the study of the CRI and Traube-Hering-Mayer oscillations.[4,5,11,19,20]

Mathematically, for the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations to derive from “The Tide,” “The Tide” must be the fundamental frequency, represented typically as $\omega 0i$ (where i = 1, 2, …, 6). In examining the results of the analysis of the data as summarized in the table above, the fundamental frequencies for the very-low-frequency range are a single value of 12 cpm. The fundamental frequencies for the low-frequency range vary from 12 cpm to 60 cpm. The fundamental frequencies for the high-frequency range vary from 6 cpm to 30 cpm. Thus, the fundamental frequencies for “The Tide” are to be found in the range of 6 cpm to 60 cpm, if indeed the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations to derive from “The Tide.”

It is evident that the fundamental frequencies range beyond the CRI, as reported by several researchers, and especially as demonstrated by Nelson and associates, Henley and associates,[37] and Moskalenko and associates. It is also evident from the analysis of data above that the fundamental frequencies associated with high-frequency range “cross” the values of the very-low frequency and low-frequency range. Intuitively, it would be appropriate for the values and ranges of the fundamental frequencies to be ordinal – that is, the fundamental frequency associated with the very-low-frequency range should be less than the fundamental frequencies associated with the low-frequency range, which, in turn, should be less than the fundamental frequencies associated with the high-frequency range. Clearly, this is not the case.

Even if we only consider the notion that the CRI, pulmonary respiration, and the Traube-Hering Mayer oscillations are due to the very-low-frequency and low-frequency ranges, with the CRI being in the “new normative range of 2-7 cpm” as reported by Nelson and associates above,[18] then the fundamental frequency would be in the range of 12 cpm to 60 cpm. Again, for the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations to be harmonics and derive from fundamental frequencies, these fundamental frequencies would have to be of smaller magnitude (that is, lesser value) than the harmonics from which they derive. Clearly, again this is not the case.

For the sake of argument, if we only consider the narrow very-low-frequency and minimal values in the low-frequency ranges, we would get a very narrow subset not demonstrated by the results reported by Nelson and associates, Henley and associates, and Moskalenko and associates cited earlier. Furthermore, we would find that the fundamental frequency is coincident with the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations that derive from it and are its harmonics. This does not fit with the mathematics of spectral analysis or Fourier series. It would also not be separately palpable, as Magoun found “[f]our definite motions have been observed at operation.”[3]

From the analysis of the data and the discussion above, it is not mathematically possible for “The Tide” to be in the range of the fundamental frequencies of the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations that derive from it and are its harmonics.

Conclusions

It is especially important to note that since this thesis was first proposed and approved in 2007, and then again proposed and approved in 2013, no work has been performed or presented by other investigators. It remains original in its concept and its contribution to the body of knowledge.

This study was limited in a number of ways. It only considers mathematical data previously reported in the literature. While it does examine the question from the purely mathematical analysis involving spectral analysis, Fourier series, harmonics, and Fourier analysis, it does not consider mathematical analysis of amplitude. Amplitude is mathematically independent of frequency, and therefore the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations do not derive from and are not harmonics of amplitude. Clinically, a low amplitude may make it difficult for the practitioner to palpate either “a wave not related to either heart rate or respiration but one which constantly maintains its own cycle” or “an undulating pulsation which has not been identified.”[3(pp 23-24)]

As stated above, it is not mathematically possible for “The Tide” to be in the range of the fundamental frequencies of the CRI, pulmonary respiration, and the Traube-Hering-Mayer oscillations that derive from it and are its harmonics. This mathematical analysis cannot be construed as negating the existence of “The Tide,” rather merely eliminates these fundamental frequencies as being “The Tide.” “The Tide” must be something else. More research is needed.

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